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On *U*-Recurrent Finsler Space

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***Abstract* - In the present paper, we introduce a Finsler space which $hv-$curvature tensor satisfies the recurrence property in sense of Cartan. The relationship between $hv-$curvature tensor U^i_{jkh} and Douglas tensor D^i_{jkh} have been studied. We obtain the necessary and sufficient condition for some tensors to be recurrent. Finally, the recurrence property in a projection on indicatrix with respect to Cartan connection has been discussed.**

Keywords: U^h-recurrent space, Douglas tensor, projection on indicatrix.

I. INTRODUCTION

Finsler geometry is usually considered as a generalization of Riemannian geometry. In fact, Riemann suggested a possibility of studying a geometry more general than Riemannian geometry. The definition for normal projective tensor N^i_{jkh} and connection coefficients Π^i_{jk} for it introduced by Yano [11, 12]. The definition for Douglas tensor and some types of it studied by Bácso and matsumoto [15]. Mishra and Lodhi [7] discussed the properties C^h-recurrent and C^v-recurrent Finsler spaces, Mohammed [5] and Hussien [14] studied the recurrence property for Cartan's second curvature tensor and Cartan's fourth curvature tensor, respectively. Alaa et al. [2] obtained the necessary and sufficient condition for some tensors to be recurrent in $G(\mathrm{B}P)-RF_n$. Additionally, the $\mathrm{B}C-$recurrent space introduced by Alaa et al. [3].

Alaa et al. [1] and Qasem and Abdallah [9] dicussed the projection on indicatrix for some tensors which satisfy the recurrence property with respect to Berwald connection, Hanballa [4] studied the projection on indicatrix for some tensors with respect to Cartan's connection.

Let F_n be an $n-$dimensional Finsler space equipped with the metric function $F(x,y)$ satisfying the request conditions [10].
The unit vector l^i and the associative vector l_i with the direction of y^i are given by

(1.1) a) $l^i = y^i/F$ and b) $l_i = y_i/F$.

Cartan $h-$*covariant differentiation* (*Cartan*'s *second kind covariant differentiation*) with respect to x^l is given by [10]

$$X^i_{|l} := \partial_l X^i - (\dot{\partial}_r X^i) G^r_l + X^r \Gamma^{*i}_{rl} .$$

For an arbitrary vector field x^l, the $h-$covariant differentiation with respect to x^l which defined above, commute with the partial differentiation with respect to y^j according to

(1.2) $\dot{\partial}_j (X^i_{|l}) - (\dot{\partial}_j X^i)_{|l} = X^r (\dot{\partial}_j \Gamma^{*i}_{rk}) - (\dot{\partial}_r X^i) P^r_{jl}$,

where $P^r_{jl} = (\dot{\partial}_j \Gamma^{*r}_{hl}) y^h = \Gamma^{*r}_{jhl} y^h$.

The $h-$covariant derivative of the vector y^i and the metric function F are vanish identically*i.e.*

(1.3) a) $y^i_{|k} = 0$ and b) $F_{|l} = 0$.

II. PRELIMINARIES

In this section, we introduced some definitions which be needed in this paper.

The normal projective tensor N^i_{jkh} is defined as follows [12]

$$N^i_{jkh} = \dot{\partial}_j \Pi^i_{kh} + \Pi^i_{rjh} \Pi^r_{ks} y^s + \Pi^i_{rh} \Pi^r_{kj} - k \mid h ,$$

where

$$\Pi^i_{jkh} = G^i_{jkh} - \frac{1}{n+1}\left(\delta^i_j G^r_{jkr} + y^i G^r_{jkhr}\right)$$

and $\Pi^i_{jkh} = \dot{\partial}_j \Pi^i_{kh}$,

Π^i_{jkh} consider the components of a tensor.

The normal projective connection coefficients Π^i_{jk} is positively homogeneous of degree zero in y^i and symmetric in their lower indices is defined by [12]

(2.1) $\Pi^i_{jk} = G^i_{jk} - y^i G^r_{jkr}$,

where $G^i_{jk} = \dot{\partial}_j G^i_k$.

Yano denoted for the tensor Π^i_{jkh} by U^i_{jkh} which is defined by [12]

(2.2) $U^i_{jkh} = G^i_{jkh} - \dfrac{1}{n+1}\left(\delta^i_j G^r_{jkr} + y^i G^r_{jkhr}\right)$

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and

(2.3) $G^r_{jkhr} = \dot{\partial}_j G^r_{khr}$,

where G^i_{jkh} consider a connection of the curvature tensor U^i_{jkh}. This tensor is homogeneous of degree -1 in y^i and symmetric in its last two indices, i.e.

$U^i_{jkh} = U^i_{jhk}$.

Also, this tensor satisfies the following

(2.4) a) $U^r_{jrh} = U^r_{jkr} = G^r_{jkr}$, b) $U^i_{jkh} y^j = 0$

and c) $U^i_{jkh} y^h = U^i_{jhk} y^h = U^i_{jk}$,

where the torsion tensor U^i_{jk} satisfies

(2.5) a) $U^i_{jk} = U^i_{kj}$, b) $U^r_{jr} = G^r_{kr}$

and c) $U^i_{jk} y^k = U^i_{kj} y^k = G^i_j$,

where the tensor G^i_j is deviation tensor which be homogeneous of degree 1 in y^i and satisfy

(2.6) $G^i_j y^j = 2G^i$,

where G^i is positively homogeneous of degree 2 in y^i.

The $U-$Ricci tensor U_{jk} satisfies the following [10]

(2.7) a) $U^r_{rkh} = U_{kh}$ and b) $U_{jk} = \frac{2}{n+1} G_{jk}$,

where the tensor G_{jk} is components of the projective connection coefficients.

The Douglas tensor is given by [6, 15]

(2.8) $D^i_{jkh} = U^i_{jkh} - \frac{1}{2}\left(\delta^i_j U_{kh} + \delta^i_k U_{jh}\right)$.

Which satisfies the following

(2.9) $D^i_{jkh} y^j = D^i_{kjh} y^j = D^i_{khj} y^j = 0$.

Definition 2.1 Let the current coordinates in the tangent space at the point x_0 be x^i, then the indicatrix I_{n-1} is a hypersurface defined by [1, 16]

$F\left(x_0\ , x^i\right) = 1$ or by the parametric form defined by $x^i = x^i\left(u^a\right)$, $a = 1, 2, \ldots, n-1$.

Definition 2.2 The projection of any tensor T^i_j on indicatrix I_{n-1} is given by [1, 4]

(2.10) $p\ .T^i_j = T^a_b\, h^i_a\, h^b_j$,

where

(2.11) $h^i_c = \delta^i_c - l^i l_c$.

The projection of the vector y^i, the unit vector l^i and the metric tensor g_{ij} on the indicatrix are given by [13]

$p.y^i = 0$, $p.l^i = 0$ and $p.g_{ij} = h_{ij}$,

where $h_{ij} = g_{ij} - l_i l_j$.

III. U^h –RECURRENT SPACE

In this section, we introduce a Finsler space which U^i_{jkh} be recurrent in sense of Cartan. Also, we find the condition for some tensors which satisfy the recurrence property.

Definition 3.1 A Finsler space F_n which the tensor U^i_{jkh} satisfies the recurrence property, i.e. characterized by

(3.1) $U^i_{jkh|l} = \lambda_l U^i_{jkh}$, $U^i_{jkh} \neq 0$,

will be called a U^h *–recurrent space* and denoted for it briefly by $U^h - RF_n$, where λ_l is the recurrence vector.

Let us consider a $U^h - RF_n$.

Transvecting (3.1) by y^i, using (1.3) and (2.4c), we get

(3.2) $U^i_{jk|l} = \lambda_l U^i_{jk}$.

Contracting the indices i and j in (3.1) and using (2.7a), we get

(3.3) $U_{kh|l} = \lambda_l U_{kh}$.

Contracting of the indices i and h in (3.1), then in view of (2.4a), we get

(3.4) $G^r_{jkr|l} = \lambda_l G^r_{jkr}$.

Contracting the indices i and k in (3.2) and using (2.5b), we get

(3.5) $G^r_{jr|l} = \lambda_l G^r_{jr}$.

In view of (2.7b) and (3.3), we get

(3.6) $G_{kh|l} = \lambda_l G_{kh}$.

Transvecting (3.2) by y^k, using (1.3) and (2.5c), we get

(3.7) $G^i_{j|l} = \lambda_l G^i_j$.

Transvecting (3.7) by y^j, using (1.3) and (2.6), we get

(3.8) $G^i_{|l} = \lambda_l G^i$.

From previous equations, we can conclude

Theorem 3.1 *The torsion tensor U^i_{jk}, $U-$Ricci tensor U_{kh}, tensor G^r_{jkr}, torsion tensor G^r_{jr}, Ricci tensor G_{kh}, deviation tensor G^i_j and curvature vector G^i behave as recurrent in $U^h - RF_n$.*

Differentiating (2.8) covariantly with respect to x^l in the sense of Cartan, we get

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(3.9) $D^i_{jkh|l} = U^i_{jkh|l} - \frac{1}{2}\left(\delta^i_j U_{kh|l} + \delta^i_k U_{jh|l}\right)$.

Using (3.1) and (3.3) in (3.9), we get

$$D^i_{jkh|l} = \lambda_l\left\{U^i_{jkh} - \frac{1}{2}\left(\delta^i_j U_{kh} + \delta^i_k U_{jh}\right)\right\}.$$

Using (2.8) in above equation, we get

(3.10) $D^i_{jkh|l} = \lambda_l D^i_{jkh}$.

Thus, we conclude

Theorem 3.2 *In $U^h - RF_n$, the Douglas tensor D^i_{jkh} is recurrent.*

If the Douglas tensor D^i_{jkh} and $U-$Ricci tensor U_{kh} are recurrent in Finsler space, then this space is necessaryto be $U^h - RF_n$. This will be seen as follows:

Equation (3.9) can be written as

(3.11) $U^i_{jkh|l} = D^i_{jkh|l} + \frac{1}{2}\left(\delta^i_j U_{kh|l} + \delta^i_k U_{jh|l}\right)$.

From (3.10) and (3.3), we have the Douglas tensor D^i_{jkh} and $U-$Ricci tensor U_{kh} behave as recurrent, then above equation become as

$$U^i_{jkh|l} = \lambda_l\left\{D^i_{jkh} + \frac{1}{2}\left(\delta^i_j U_{kh} + \delta^i_k U_{jh}\right)\right\}.$$

Using (2.8) in above equation, we get

$$U^i_{jkh|l} = \lambda_l U^i_{jkh}.$$

Thus, we conclude

Theorem 3.3 *In Finsler space F_n, if the Douglas tensor and $U-$Ricci tensor are recurrent, then this space is necessarilyconsidered $U^h - RF_n$.*

Differentiating (3.4) partially with respect to y^h, we get

$$\dot{\partial}_h G^r_{jkr|l} = \left(\dot{\partial}_h \lambda_l\right) G^r_{jkr} + \lambda_l\left(\dot{\partial}_h G^r_{jkr}\right).$$

Using commutation formula exhibited by (1.2) for G^r_{jkr} and (2.3) in above equation, we get

$$G^r_{jkhr|l} - G^r_{skr}\left(\dot{\partial}_h \Gamma^{*s}_{jl}\right) - G^r_{jsr}\left(\dot{\partial}_h \Gamma^{*s}_{kl}\right) - G^r_{sjkr} P^s_{hl} = \left(\dot{\partial}_h \lambda_l\right) G^r_{jkr} + \lambda_l G^r_{jkhr}.$$

Therefore

(3.12) $G^r_{jkhr|l} = \lambda_l G^r_{jkhr}$

if and only if

(3.13) $G^r_{skr}\left(\dot{\partial}_h \Gamma^{*s}_{jl}\right) + G^r_{jsr}\left(\dot{\partial}_h \Gamma^{*s}_{kl}\right) + \left(\dot{\partial}_h \lambda_l\right) G^r_{jkr} + G^r_{sjkr} P^s_{hl} = 0$.

Differentiating (2.2) covariantly with respect to x^l in the sense of Cartan and using (1.3), we get

$$U^i_{jkh|l} = G^i_{jkh|l} - \frac{1}{n+1}\left(\delta^i_j G^r_{jkr|l} + y^i G^r_{jkhr|l}\right).$$

Using (3.1) in above equation, then using (2.2), we get

$$G^i_{jkh|l} - \frac{1}{n+1}\lambda_l\left(\delta^i_j G^r_{jkr} + y^i G^r_{jkhr}\right) + \frac{1}{n+1}\left(\delta^i_j G^r_{jkr|l} + y^i G^r_{jkhr|l}\right) = \lambda_l G^i_{jkh}.$$

Using (3.4) and (3.12) in above equation, we get

(3.14) $G^i_{jkh|l} = \lambda_l G^i_{jkh}$.

From equation (3.12) and (3.14), we get

Theorem 3.4 *The tensors G^r_{jkhr} and G^i_{jkh} in $U^h - RF_n$ behave as recurrent if and only if (3.13) holds.*

IV. PROJECTION ON INDICATRIX IN SENSE OF CARTAN

In this section, we prove that, if the tensors behave as recurrent, then the projection of them are recurrent $U^h - RF_n$. Also we find the condition for the projection of some tensors on Indicatrix be recurrent.

Let us consider a $U^h - RF_n$.

We know that, the curvature tensor U^i_{jkh} behaves as recurrent *i.e.* satisfies (3.1). Now, in view of (2.10), the projection of the curvature tensor U^i_{jkh} on indicatrix is given by

(4.1) $p.U^i_{jkh} = U^a_{bcd} h^i_a h^b_j h^c_k h^d_h$.

Taking covariant derivative of (4.1) with respect to x^l in sense of Cartan, using (3.1) and the fact that $h^i_{j|l} = 0$, we get

$$\left(p.U^i_{jkh}\right)_{|l} = \lambda_m U^a_{bcd} h^i_a h^b_j h^c_k h^d_h.$$

Using (4.1) in above equation, we get

(4.2) $\left(p.U^i_{jkh}\right)_{|l} = \lambda_m\left(p.U^i_{jkh}\right)$.

Thus, we conclude

Theorem 4.1 *If the curvature tensor U^i_{jkh} behaves as recurrent, then the projection of itin $U^h - RF_n$ on indicatrix is recurrent in sense of Cartan.*

We know that, the torsion tensor U^i_{jk} behaves as recurrent *i.e.* satisfies (3.2). In view of (2.10), the projection of the torsion tensor U^i_{jk} on indicatrix is given by

(4.3) $p.U^i_{jk} = U^a_{bc} h^i_a h^b_j h^c_k$.

Taking covariant derivative of (4.3) with respect to x^l in sense of Cartan, using (3.2) and the fact that $h^i_{j|l} = 0$, we get

$$\left(p.U^i_{jk}\right)_{|l} = \lambda_l U^a_{bc} h^i_a h^b_j h^c_k.$$

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Using (4.3) in above equation, we get

(4.4) $\left(p.U^i_{jk}\right)_{|l} = \lambda_l \left(p.U^i_{jk}\right)$.

Thus, we conclude

Theorem 4.2 *If the torsion tensor U^i_{jk} behaves as recurrent, then the projection of itin $U^h - RF_n$ on indicatrix is recurrent in sense of Cartan.*

We know that, the $U-$Ricci tensor U_{kh} behaves as recurrent *i.e.* satisfies (3.3). In view of (2.10), the projection of the $U-$Ricci tensor U_{kh} on indicatrix is given by

(4.5) $p.U_{kh} = U_{ab} h^a_k h^b_h$.

Taking covariant derivative of (4.5) with respect to x^l in sense of Cartan, using (3.3) and the fact that $h^i_{j|l} = 0$, we get

$$\left(p.U_{kh}\right)_{|l} = \lambda_l U_{ab} h^a_k h^b_h.$$

Using (4.5) in above equation, we get

(4.6) $\left(p.U_{kh}\right)_{|l} = \lambda_l U_{ab} h^a_k h^b_h$.

Thus, we conclude

Theorem 4.3 *If the $U-$Ricci tensor U_{kh} behaves as recurrent, then the projection of itin $U^h - RF_n$ on indicatrix is recurrent in sense of Cartan.*

We know that, the Dougles tensor D^i_{jkh} behaves as recurrent *i.e.* satisfies (3.10). In view of (2.10), the projection of the Dougles tensor D^i_{jkh} on indicatrix is given by

(4.7) $p.D^i_{jkh} = D^a_{bcd} h^i_a h^b_j h^c_k h^d_h$.

Taking covariant derivative of (4.7) with respect to x^l in sense of Cartan, using (3.10) and the fact that $h^i_{j|l} = 0$, we get

$$\left(p.D^i_{jkh}\right)_{|l} = \lambda_l D^a_{bcd} h^i_a h^b_j h^c_k h^d_h.$$

Using (4.7) in above equation, we get

(4.8) $\left(p.D^i_{jkh}\right)_{|l} = \lambda_l \left(p.D^i_{jkh}\right)$.

Thus, we conclude

Theorem 4.4 *If the Douglas tensor D^i_{jkh} behaves as recurrent, then the projection of itin $U^h - RF_n$ on indicatrix is recurrent in sense of Cartan.*

We know that the projection of the curvature tensor U^i_{jkh} on indicatrix behaves as recurrent *i.e.* satisfied (4.2).

Using (2.10) in (4.2), we get

$$\left(U^a_{bcd} h^i_a h^b_j h^c_k h^d_h\right)_{|l} = \lambda_l U^a_{bcd} h^i_a h^b_j h^c_k h^d_h.$$

Using (2.11) in above equation, we get

$$\begin{aligned}
&(U^i_{jkh} - U^i_{jkd} l^d l_h - U^i_{jch} l^c l_k + U^i_{jcd} l^c l_k l^d l_h - U^i_{bkh} l^b l_j \\
&+ U^i_{bkd} l^b l_j l^d l_h + U^i_{bch} l^b l_j l^c l_k - U^i_{bcd} l^b l_j l^c l_k l^d l_h \\
&- U^a_{jkh} l^i l_a + U^a_{jkd} l^i l_a l^d l_h + U^a_{jch} l^i l_a l^c l_k - U^a_{jcd} l^i l_a l^c l_k l^d l_h \\
&+ U^a_{bkh} l^i l_a l^b l_j - U^a_{bkd} l^i l_a l^b l_j l^d l_h - U^a_{bch} l^i l_a l^b l_j l^c l_k + U^a_{bcd} l^i l_a l^b l_j l^c l_k l^d l_h)_{|l} \\
&= \lambda_l (U^i_{jkh} - U^i_{jkd} l^d l_h - U^i_{jch} l^c l_k + U^i_{jcd} l^c l_k l^d l_h - U^i_{bkh} l^b l_j \\
&+ U^i_{bkd} l^b l_j l^d l_h + U^i_{bch} l^b l_j l^c l_k - U^i_{bcd} l^b l_j l^c l_k l^d l_h \\
&- U^a_{jkh} l^i l_a + U^a_{jkd} l^i l_a l^d l_h + U^a_{jch} l^i l_a l^c l_k - U^a_{jcd} l^i l_a l^c l_k l^d l_h \\
&+ U^a_{bkh} l^i l_a l^b l_j - U^a_{bkd} l^i l_a l^b l_j l^d l_h - U^a_{bch} l^i l_a l^b l_j l^c l_k + U^a_{bcd} l^i l_a l^b l_j l^c l_k l^d l_h)
\end{aligned}$$

Using (1.1a) in above equation, then using (2.4b) and (2.4c) in the resulting equation, we get

$$\begin{aligned}
&(U^i_{jkh} - \frac{1}{F} U^i_{jk} l_h - \frac{1}{F} U^i_{jh} l_k + \frac{1}{F^2} G^i_j l_k l_h \\
&- U^a_{jkh} l^i l_a + \frac{1}{F} U^a_{jk} l^i l_a l_h + \frac{1}{F} U^a_{jh} l^i l_a l_k - \frac{1}{F^2} G^a_j l^i l_a l_k l_h)_{|l} \\
&= \lambda_l (U^i_{jkh} - \frac{1}{F} U^i_{jk} l_h - \frac{1}{F} U^i_{jh} l_k + \frac{1}{F^2} G^i_j l_k l_h \\
&- U^a_{jkh} l^i l_a + \frac{1}{F} U^a_{jk} l^i l_a l_h + \frac{1}{F} U^a_{jh} l^i l_a l_k - \frac{1}{F^2} G^a_j l^i l_a l_k l_h).
\end{aligned}$$

Now, since the tensors U^i_{jk} and G^i_j are recurrent, i.e. satisfy (3.2) and (3.7), respectively. Then by using (1.1) and (1.3) in above equation, we have

(4.9) $\left(U^i_{jkh} - U^a_{jkh} l^i l_a\right)_{|l} = \lambda_l (U^i_{jkh} - U^a_{jkh} l^i l_a)$.

Thus, we conclude

Theorem 4.5 *If the projection of the tensor $\left(U^i_{jkh} - U^a_{jkh} l^i l_a\right)$ on indicatrix is recurrent, then the space is $U^h - RF_n$, provided U^i_{jk}* and *G^i_j are recurrent in sense of Cartan.*

From (4.9), we can get

Corollary 4.1 *In $U^h - RF_n$, the projection of the tensor U^i_{jkh} on indicatrix is recurrent, if and only if $U^a_{jkh} l_a$ is recurrent.*

We know that the projection of the torsion tensor U^i_{jk} on indicatrix behaves as recurrent *i.e.* satisfied (4.4).

Using (2.10) in (4.4), we get

$$\left(U^a_{bc} h^i_a h^b_j h^c_k\right)_{|l} = \lambda_l U^a_{bc} h^i_a h^b_j h^c_k.$$

Using (2.11) in above equation, we get

$$\begin{aligned}
&(U^i_{jk} - U^i_{jc} l^c l_k - U^i_{bk} l^b l_j + U^i_{bc} l^b l_j l^c l_k - U^a_{jk} l^i l_a + U^a_{jc} l^i l_a l^c l_k \\
&+ U^a_{bk} l^i l_a l^b l_j - U^a_{bc} l^i l_a l^b l_j l^c l_k)_{|l} \\
&= \lambda_l (U^i_{jk} - U^i_{jc} l^c l_k - U^i_{bk} l^b l_j + U^i_{bc} l^b l_j l^c l_k - U^a_{jk} l^i l_a + U^a_{jc} l^i l_a l^c l_k \\
&+ U^a_{bk} l^i l_a l^b l_j - U^a_{bc} l^i l_a l^b l_j l^c l_k).
\end{aligned}$$

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Using (1.1a) in above equation, then using (2.5c) and (2.6) in the resulting equation, we get

$$(U^i_{jk} - \frac{1}{F}G^i_j l_k - \frac{1}{F}G^i_k l_j + \frac{1}{F^2}G^i l_j l^c l_k - U^a_{jk} l^i l_a + \frac{1}{F}G^a_j l^i l_a l_k + \frac{1}{F}G^a_k l^i l_a l_j - \frac{1}{F^2}G^a l^i l_a l_j l_k)_{|l}$$

$$= \lambda_l (U^i_{jk} - \frac{1}{F}G^i_j l_k - \frac{1}{F}G^i_k l_j + \frac{1}{F^2}G^i l_j l^c l_k - U^a_{jk} l^i l_a + G^a_j l^i l_a l_k + G^a_k l^i l_a l_j - \frac{1}{F^2}G^a l^i l_a l_j l_k).$$

Now, since the tensors G^i_j and G^i are recurrent, i.e. satisfy (3.7) and (3.8), respectively. Then by using (1.1) and (1.3) in above equation, we have

(4.10) $\left(U^i_{jk} - U^a_{jk} l^i l_a\right)_{|l} = \lambda_l \left(U^i_{jk} - U^a_{jk} l^i l_a\right).$

Thus, we conclude

Theorem 4.6 *If the projection of the tensor* $\left(U^i_{jk} - U^a_{jk} l^i l_a\right)$ *on indicatrix is recurrent, then the space is* $U^h - RF_n$ *, provided* G^i_j and G^i *are recurrent in sense of Cartan.*

From (4.10), we can also conclude

Corollary 4.2 *In* $U^h - RF_n$*, the projection of the torsion tensor* U^i_{jk} *on indicatrix is recurrent, if and only if* $U^a_{jk} l_a$ *is recurrent.*

We know that the projection of the $U-$Ricci tensor U_{kh} on indicatrix behaves as recurrent *i.e.* satisfied (4.6).

Using (2.10) in (4.6), we get

$$\left(U_{ab} h^a_k h^b_h\right)_{|l} = \lambda_l U_{ab} h^a_k h^b_h .$$

Using (2.11) in above equation, we get

$$\left(U_{kh} - U_{kb} l^b l_h - U_{ah} l^a l_k + U_{ab} l^a l^b l_k l_h\right)_{|l}$$
$$= \lambda_l \left(U_{kh} - U_{kb} l^b l_h - U_{ah} l^a l_k + U_{ab} l^a l^b l_k l_h\right).$$

Now, in view of (1.1) and if $U_{kb}\, y^b = 0 = U_{ah}\, y^a$, then above equation becomes

$$U_{kh|l} = \lambda_l U_{kh} .$$

Thus, we conclude

Theorem 4.7 *In* $U^h - RF_n$*, if the projection of the* $U-$*Ricci tensor* U_{kh} *on indicatrix behaves as recurrent, then the* $U-$ *Ricci tensor* U_{kh} *also behave as recurrent, provided* $U_{kb}\, y^b = 0 = U_{ah}\, y^a$.

V. CONCLUSION

We obtained the relationship between some connection coefficients for different tensors, we proved that the Douglas tensor satisfies the recurrence property. Also, we discussed the projection on indicatrix in sense of Cartan for some tensors which behaves as recurrent in $U^h - RF_n$.

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