

Flutter Stability Analysis of Carbon Nanotubes (CNT) Conveying Fluid in Time Domain Using Finite Element Method

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Abstract - Carbon Nanotube (CNT) is a material consisting of carbon atoms arranged in a series of condensed benzene rings that are coiled into a tubular structure. CNTs are used in various fields, such as reinforcing materials, coatings and films, biotechnology and biomedicine, and nano electromechanic systems (NEMS). The application of CNT in the biotechnology and biomedical fields as a nanopipe for carrying fluids is due to its high elastic modulus, good thermal and electrical conductivity, and other superior mechanical properties. One of the phenomena that can reduce CNT performance is flutter, so fluid-structure interaction is one of the important considerations in the design and application of CNTs. This study analyzes the effect of mass ratio on flutter instability due to flow in the CNT cantilever using the finite element method. Meanwhile, the system's responses are calculated using the Newmark method time scheme. The structural characteristics of the CNT were modeled as the Euler-Bernoulli beam. In this case, the fluid flowing in it is characterized in terms of two parameters, namely the average flow velocity (U) and the fluid mass density (per unit length). The results showed that the analysis of the response in the time domain is very sensitive to changes in flow velocity, the critical speed will decrease with an increase in the aspect ratio. and the mass ratio affects the flutter stability limit, where the larger the mass ratio for a given fluid density, the higher the flutter stability limit. The findings also revealed that the higher the density of the fluid flowing through the CNT, the lower its ultimate stability limit.

Keywords: carbon nanotube, finite element method, flutter, time response.

I. INTRODUCTION

The stability of the mass-carrying structure has been studied by scientists for many years. This research focuses on the stability of the elastic pipe conveying fluid. When the fluid flows in the elastic pipe there is a dynamic interaction between

the fluid and the pipe causing energy to be transferred from the fluid to the pipe. It will cause the pipe stability will be lost if the energy exceeds its capacity [1].

One of the mass carrier structures in the biomechanics is carbon nanotubes (CNT). Carbon nanotubes (CNT) are the third allotropic form of carbon-fullerene rolled into cylindrical tubes. To be integrated into biological systems, CNTs can be chemically modified or functionalized with therapeutically active molecules by forming stable covalent bonds or supramolecular assemblies based on noncovalent interactions. Due to their high carrying capacity, biocompatibility and cell specificity, various cancer cells have been explored with CNTs for the evaluation of pharmacokinetic parameters, cell viability, cytotoxicity and drug delivery in tumor cells [2][3][4].

With its superior mechanical properties, CNT is very well suited for conveying fluids [5]. Therefore CNT is a suitable delivery tool for various kinds of therapeutic molecules (drugs, proteins, enzymes, genes, etc.) into the body without being affected by metabolic processes [2]. In terms of disease detection and diagnosis, CNT can be used as a means of transporting biosensors. For example, many researchers combine glucose-oxidase with CNT in monitoring blood sugar in diabetic patients [3][4]. CNT also opens up new alternatives in drug delivery systems because functional CNTs are able to penetrate cell membranes so that drugs can work directly in target cells [2].

One of the most widely used uses of CNTs in drug delivery is in the treatment of cancer. The effectiveness of anticancer drugs is hampered by their toxicity to healthy cells and limited cellular penetration ability. This problem is resolved by the functional ability of CNTs to specifically identify cell receptor surfaces when interacting with cancer cells, and then CNTs can pass through the cytoplasmic and nuclear membranes by endocytosis [2]. Several anticancer drugs conjugated with functional CNT have been tested in

vitro and in vivo, including epirubicin, doxorubicin, cisplatin, methotrexate, quercetin, and paclitaxel [3].

In this research, the effect of internal fluid flow on free vibration and instability has been studied in CNTs that have pinned or clamped supports at both ends [3]. In that case, the vibrations caused by the fluid flow inside the CNT are periodic vibrations with constant amplitude, and when the critical velocity of the fluid is reached, the lowest frequency will decrease to zero. This causes divergent instability in the CNT, similar to static buckling or compressed elastic. However, many applications of CNTs as nanopipes involve cantilever structures, where one end has a fixed support and the other end is a free end. In contrast to other types of supports, it is known that vibrations caused by fluid flow in the cantilever pipe are not conservative. This is indicated by the amplitude, which can increase or decrease over time. When the flow velocity is low enough, the vibration of the cantilever pipe will gradually decrease and fade. However, when the critical speed is reached, the amplitude will increase. This is when a phenomenon called “flutter” occurs.

Flutter is generally defined as a self-excited vibrational instability due to the interaction between the structure and the fluid flow [6]. When flutter occurs, the fluid force that previously provided damping actually provides additional energy to the structure, resulting in a larger amplitude. The increased amplitude will then increase the fluid force again, and so on. In the CNT case, flutter can occur due to the fluctuating force that arises due to the fluid flow in the CNT pipe [7][8][9]. These forces excite the pipe to vibrate and can make it unstable. Flutter is a dynamic instability in the field of fluid-structure interaction (FSI) which often results in structure failure, so this phenomenon is undesirable [1][6].

Because flutter is an important thing that needs to be considered in the application and design of CNT, this research was conducted to study the vibration of CNT cantilever pipes caused by fluid flow and the flutter instability that will occur. The characteristics of the CNT structure are modeled by the Euler-beam model. In this case, the fluid flowing in it is characterized by two parameters, namely the average flow velocity (U) and the mass density of the fluid (per unit length).

II. METHODOLOGY

2.1 CNT Cantilever Modeling

As shown in Figure 1, CNT is described as a cantilever pipe, and the type of CNT used is MWCNT (multiwall carbon nanotube). In this study, CNT is considered to have a foundation with spring stiffness per unit length represented by the Winkler constant (K). To facilitate further analysis, several assumptions are made, namely that the flowing fluid is an

incompressible fluid, in a steady-state condition, and has a uniform velocity.

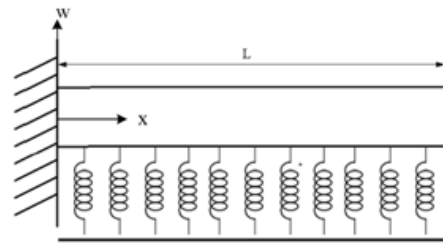


Figure 1: Cantilever CNT model

2.2 Mathematical Formulation

Assuming that the flowing fluid is an incompressible fluid, in a steady-state condition, has a uniform velocity, and is supported along its span with an elastic winker type foundation involving distributed spring stiffness K per unit length, the winker value must be added to the equation of motion. Assuming the influence of gravity and the axial displacement of the cantilever pipe are neglected, the equation of motion can be written as follows [1][6][10]:

$$EI \frac{\partial^4 w}{\partial x^4} + m \frac{\partial^2 w}{\partial t^2} + \rho A \left[\frac{\partial^2 w}{\partial t^2} + 2U \frac{\partial^2 w}{\partial x \partial t} + U^2 \frac{\partial^2 w}{\partial x^2} \right] + Kw = 0 \quad (1)$$

Where E , I , m , w and L represent the modulus of elasticity, moment of inertia of cross-sectional area, mass per unit length, transverse deflection and length of the pipe, respectively. While ρ , U and A respectively represent the density, flow rate and cross-sectional area of the fluid flow.

2.3 Finite Element Modeling

In this work, polynomial with cubic function is used to describe the displacement of the beam element. The Hermite cubic polynomial function is written as [11]:

$$w_e(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 \quad (2)$$

The unknown nodal displacement at the ends of each element is given as

$$\{d_e\}^T = \{d_1^e \quad d_2^e \quad d_3^e \quad d_4^e\} \quad (3)$$

and the displacement function for the beam element is given by

$$w_e(x) = [N_1 \quad N_2 \quad N_3 \quad N_4] \{d_{i,e}\} \quad (4)$$

From the assumption for Euler-Bernoulli beam the shape functions in Eq. (4) are given as follows

$$\begin{aligned} N_1 &= 1 - \frac{3x_e^2}{a^2} + \frac{2x_e^3}{a^3}; & N_2 &= x_e - \frac{2x_e^2}{a} + \frac{x_e^3}{a^2}; \\ N_3 &= \frac{3x_e^2}{a^2} - \frac{2x_e^3}{a^3}; & N_4 &= -\frac{x_e^2}{a} + \frac{x_e^3}{a^2} \end{aligned} \quad (5)$$

Where a indicates length of element. Application of Hermitian shape function and Galerkin’s method result stiffness and mass element matrices of the structure as follow

$$[K_s^e] = \int_0^a [N'']^T EI [N''] dx_e$$

$$= \frac{EI}{a^3} \begin{bmatrix} 12 & 6a & -12 & 6a \\ 6a & 4a^2 & -6a & 2a^2 \\ -12 & -6a & 12 & -6a \\ 6a & 2a^2 & -6a & 4a^2 \end{bmatrix} \quad (6)$$

$$[M_s^e] = \int_0^a m [N]^T [N] dx_e$$

$$= \frac{ma}{420} \begin{bmatrix} 156 & 22a & 54 & -13a \\ 22a & 4a^2 & 13a & -3a^2 \\ 54 & 13a & 156 & -22a \\ -13a & 3a^2 & -22a & 4a^2 \end{bmatrix} \quad (7)$$

Whereas the matrix of fluid element mass, fluid element damping and fluid element stiffness are respectively written as follow

$$[M_f^e] = \int_0^a \rho A [N]^T [N] dx_e$$

$$= \frac{\rho A a}{420} \begin{bmatrix} 156 & 22a & 54 & -13a \\ 22a & 4a^2 & 13a & -3a^2 \\ 54 & 13a & 156 & -22a \\ -13a & 3a^2 & -22a & 4a^2 \end{bmatrix} \quad (8)$$

$$[C_f^e] = 2 \int_0^a \rho A U [N]^T [N'] dx_e - \rho A U [N]^T [N] \Big|_{x_e=0}^{x_e=a}$$

$$= \frac{\rho A U}{30} \begin{bmatrix} 0 & 6a & 30 & -6a \\ -6a & 0 & 6a & -a^2 \\ -30 & -6a & 0 & 6a \\ 6a & a^2 & -6a & 0 \end{bmatrix} \quad (9)$$

$$[K_f^e] = \int_0^a \rho A U^2 [N]^T [N''] dx_e - \rho A U^2 [N]^T [N'] \Big|_{x_e=0}^{x_e=a}$$

$$= \frac{\rho A U^2}{30a} \begin{bmatrix} -36 & -3a & 36 & -3a \\ -3a & -4a^2 & 3a & a^2 \\ 36 & 3a & -36 & 3a \\ -3a & a^2 & 3a & -4a^2 \end{bmatrix} \quad (10)$$

Furthermore the total matrix of the mass, damping and stiffness of the system element then can be read as follow, respectively

$$[M^e] = [M_s^e] + [M_f^e]; [C^e] = [C_f^e]; [K^e] = [K_s^e] + [K_f^e] \quad (11)$$

The expression in Eq. (11) above is the function of the length of each element. As a result, the length of each element is computed from the coordinate values of the nodes associated with the element. Once these matrices are computed, they need to be assembled into the system matrix.

To this end, it is needed to know where the elements of the element matrix are to be located in the system matrix.

After assembling all the element matrices and applying the boundary conditions for the free-fixed supports, the global form of the mass, damping, and stiffness matrices of the overall system can be expressed as:

$$M_{ij} = M_{(i+2)(j+2)}; C_{ij} = C_{(i+2)(j+2)}; K_{ij} = K_{(i+2)(j+2)} \quad (12)$$

While the displacement vector is expressed by

$$\{q_i\} = \{d_{i+2}\} \quad (13)$$

The equations of motion of the system thus can be expressed in the form

$$M\ddot{q} + C\dot{q} + Kq = 0 \quad (14)$$

2.4 Newmark Method Time Scheme

For solving Eq. (14) a numerical integration technique is applied. In the present study, the Newmark method time scheme was chosen for solving a system of O.D.E in structural dynamics. In this scheme, the system of O.D.E’s is fully discretized into a set of algebraic equations by using a two parameter “alpha-beta” approximation. The nodal displacements (rotations) and their derivatives are approximated in the form [12].

$$[\bar{J}]\{q\}_{s+1} = \{\bar{F}\}_{s+1} \quad (15)$$

$$[\bar{J}] = [K] + a_3[M] + a_5[C] \quad (16)$$

$$\{\bar{F}\}_{s+1} = \{F\}_{s+1} + [M](a_3\{q\}_s + a_4\{\dot{q}\}_s + a_5\{\ddot{q}\}_s) + [C](a_5\{q\}_s + a_6\{\dot{q}\}_s + a_7\{\ddot{q}\}_s) \quad (17)$$

$$\{\ddot{q}\}_{s+1} = a_3(\{q\}_{s+1} - \{q\}_s) - a_4\{\dot{q}\}_s - a_5\{\ddot{q}\}_s \quad (18)$$

$$\{\dot{q}\}_{s+1} = \{q\}_s + a_2\{\dot{q}\}_s + a_1\{\ddot{q}\}_{s+1} \quad (19)$$

$$a_1 = \alpha\Delta t, a_2 = (1 - \alpha)\Delta t, a_3 = \frac{2}{\gamma(\Delta t)^2}, a_4 = \frac{2}{\gamma\Delta t^2},$$

$$a_5 = \frac{1}{\gamma} - 1, a_6 = \frac{2\alpha}{\gamma} - 1, a_7 = \left(\frac{2\alpha}{\gamma} - 2\right)\frac{\Delta t}{2} \quad (20)$$

For solving Eq. (14) in the present study all initial conditions will be taken as zero. Equations (15)-(19) are then repeated (in a loop) for however long desired.

III. RESULTS AND DISCUSSION

Table 1 shows the CNT and fluid parameters considered in the present study as used in the study of Galanov et. al. [13]. The analysis was carried out using FEM and the time scheme of the Newmark method in order to determine the response of the system due to initial displacement. The

system's time responses are determined for various fluid flow velocities. The characteristics of the response then indicate the system's stability condition. The flutter stability onset of the system is indicated by the sinusoidal response, and the related flow velocity is called the critical speed. Meanwhile, when the response exhibits amplitude decay, the system is in a stable state for the related fluid flow velocity. Otherwise, if the response of the system becomes divergent, then the critical speed is already exceeded.

Table 1: Parameter of CNT

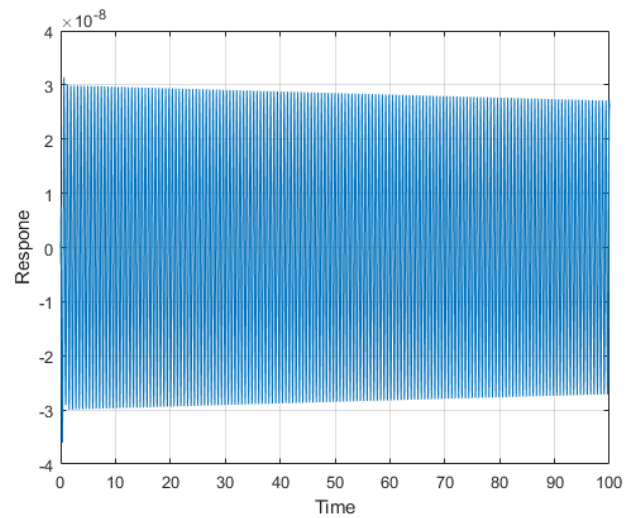
Parameter	Value
CNT Young's modulus, E	10^{12}N/m^2
Outer radius, R	40nm
Inner radius, r	20nm
Mass density of fluid, ρ	1000 Kg/m^3
Mass density of CNT, ρ_s	2300 Kg/m^3
Pipe length, L	$2 \cdot R \cdot 150$
Poisson's ratio, ν	0.3

Figs. 2–5 show the CNT response at the free end (tip response) for values of aspect ratio ($L/2R$) of 50, 100, and 150, respectively. The results of Fig. 2 indicate that flutter onset occurs at a fluid flow of 7559 m/s. This condition is characterized by a sinusoidal response amplitude, as shown in Fig. 2b. It can be seen that for a fluid flow velocity of 7558 m/s, the response amplitude will decay, which means that the system is stable (Fig. 2a), and for a flow velocity of 7560 m/s, the response amplitude becomes divergent, which means the system is unstable (Fig. 2c).

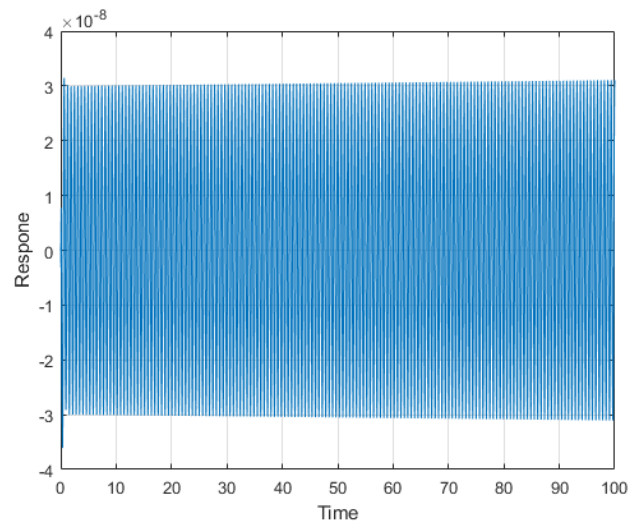
The same thing also happens to CNT with an aspect ratio value ($L/2R$) of 50, as shown in Figure 3. It can be seen from Figure 3 that for $L/2R = 50$, the flutter stability limit occurs at a fluid flow velocity of 1512 m/s, as shown in Figure 3b. Whereas for $L/2R = 100$, flutter occurs at a fluid flow velocity of 756 m/s, as shown in Figure 4b. Furthermore, as shown in Figure 5b, for $L/2R = 150$, the flutter stability limit occurs at a fluid flow velocity of 504 m/s.

Overall, the results of the flutter stability analysis of the fluid-conveying CNT are shown in Table 2. The table also includes the results of an analysis on a similar case carried out by Yoon et al. [7]. It should be noted here that Yooet. al. performed flutter analysis in the frequency domain. Nevertheless, the results obtained from this study provide

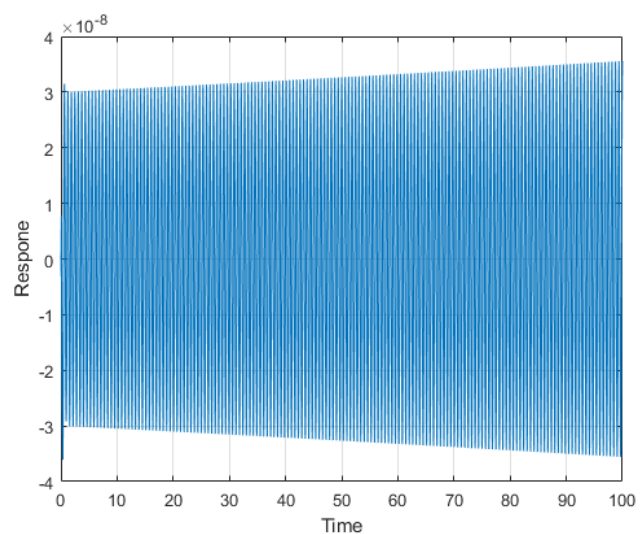
results that are very close to the results given by Yoo et. al., as shown in Table 2.



(a)

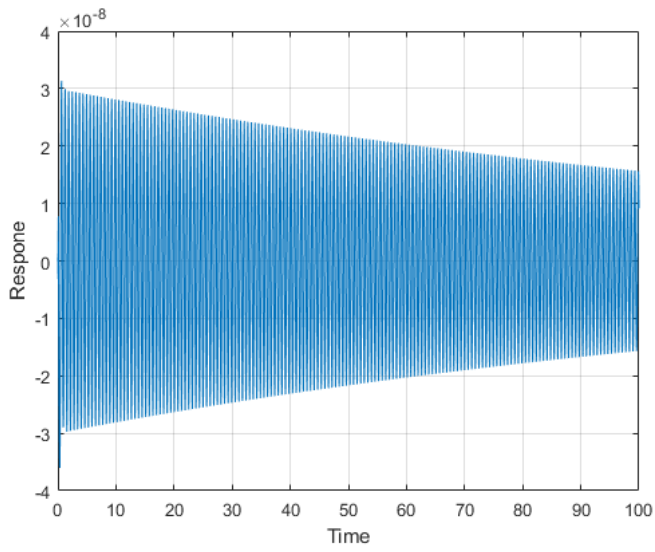


(b)

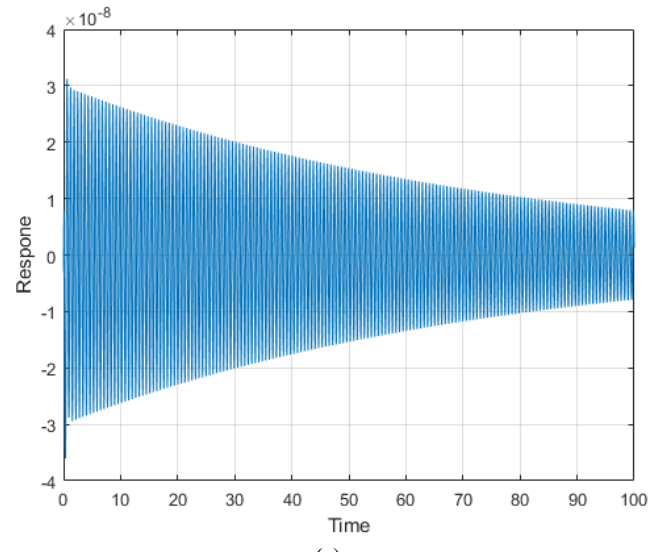


(c)

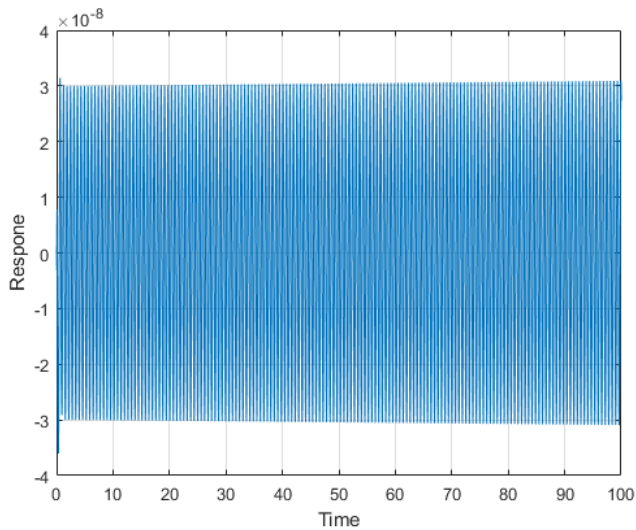
Figure 2: Tip response of CNT with $L/2R=10$ for (a) $v = 7558$ m/s, (b) $v = 7559$ m/s, (c) $v = 7560$ m/s



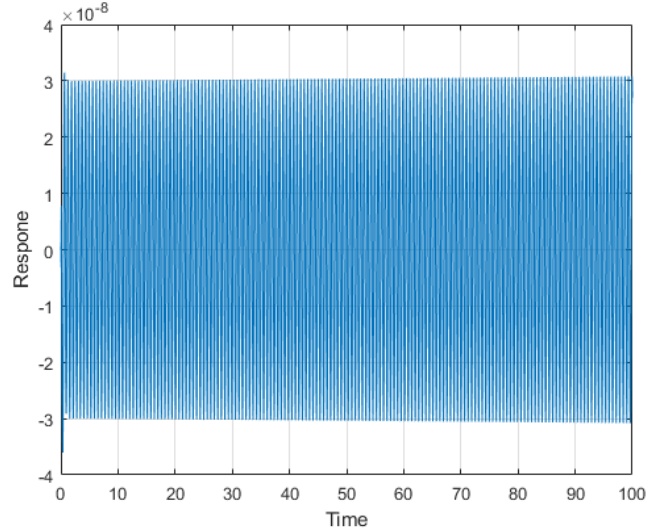
(a)



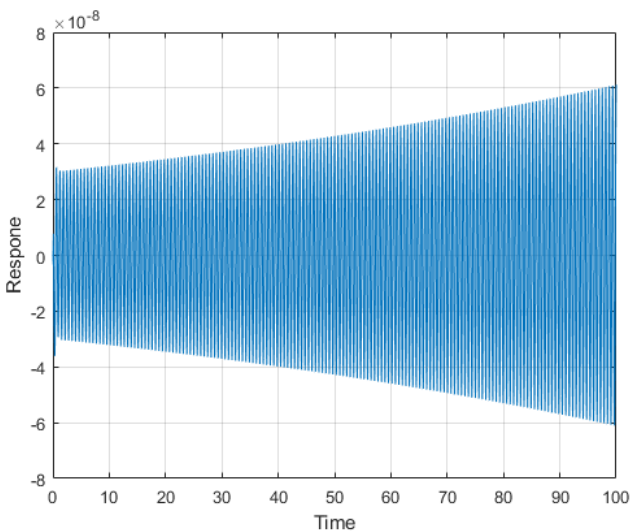
(a)



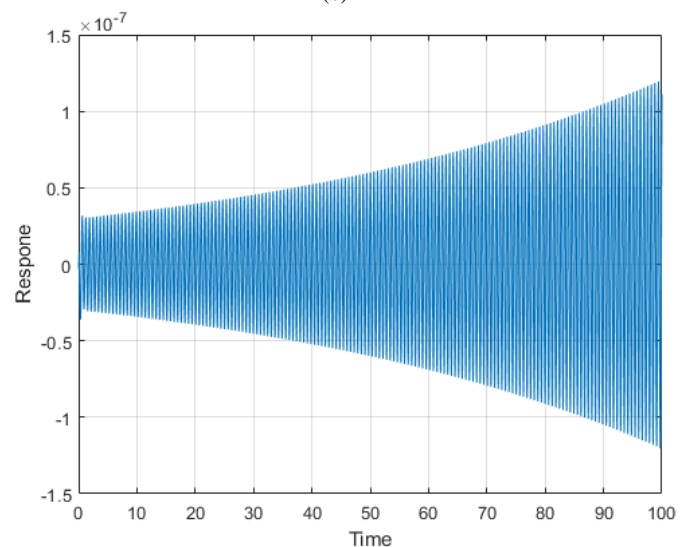
(b)



(b)



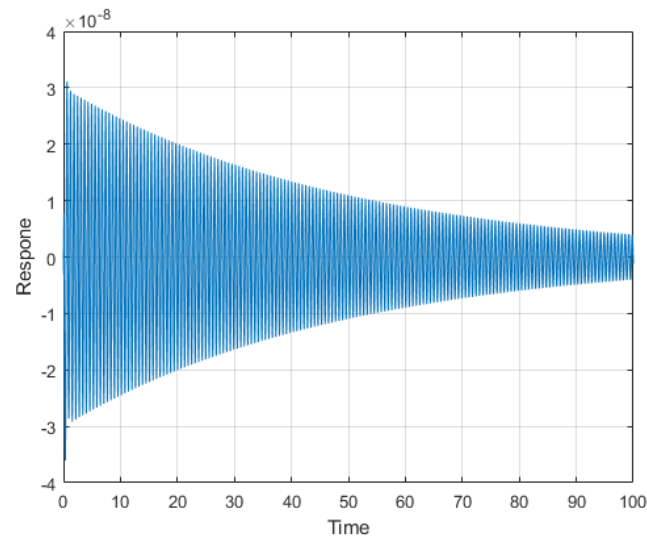
(b)



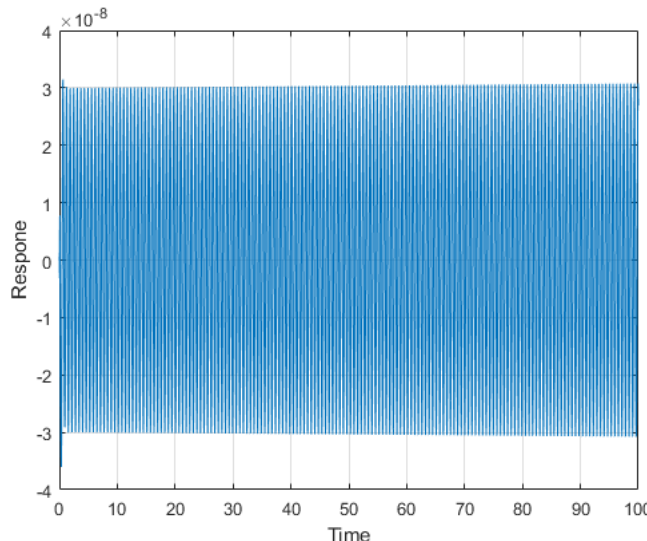
(c)

Figure 3: Tip response of CNT with $L/2R=50$ for (a) $v=1511$ m/s, (b) $v=1512$ m/s, (c) $v=1513$ m/s

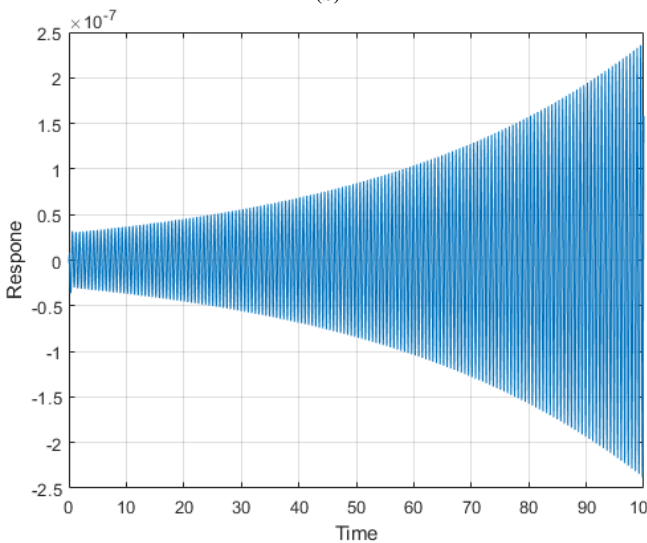
Figure 4: Tip response of CNT with $L/2R=100$ for (a) $v=755$ m/s, (b) $v=756$ m/s, (c) $v=757$ m/s



(a)



(b)



(c)

Figure 5: Tip response of CNT with $L/2R = 150$ for (a) $v = 503$ m/s, (b) $v = 504$ m/s, (c) $v = 505$ m/s

Table 2: Critical flow speed (m/s) for various aspect ratios

$L/2R$	10	50	100	150
Ref. [7]	7560	1510	750	504
Present study	7559	1512	756	504
Difference	0.01%	0.13%	0.80%	0.00%

From Table 2, it can be seen that the critical speed will decrease with an increase in the aspect ratio. This condition occurs because, with an increase in the aspect ratio, the CNT pipe will become more elastic. As a consequence, CNTs flutter more easily. In other words, the critical speed decreases.

The interesting thing that can be seen in Figures 2 - 5 is that the analysis of the response in the time domain showed that it is very sensitive to changes in flow velocity. It also appears that the sensitivity will be increasingly visible with an increase in the aspect ratio. Such conditions indicate that the determination of the flutter stability limit will be more accurate if it is carried out in the time domain.

Another thing that can be obtained from the analysis of the time domain is a visual description of the behavior of the system. This cannot be obtained if the analysis is carried out in the frequency domain.

In this research, the influence of fluid density and mass ratio on critical velocity is also studied. Table 3 shows the critical velocity of the fluid flow for various values of fluid density and mass ratio. It is very interesting to note that the fluid density has a strong effect on the critical speed of fluid flow. As the fluid density increases, the flexural frequencies decrease and thus the critical speed also decrease. As a result, the critical flow speed depends on the fluid density, which can be inferred from Table 3.

Table 3: Critical flow speed of CNT (m/s) for various fluid density and mass ratio

$\rho A/(m + \rho A)$	Fluid density, ρ (Kg/m ³)			
	1000	1500	2000	2500
0.01	437.20	357.00	309.20	276.55
0.05	456.30	372.60	322.68	288.60
0.1	485.75	396.61	343.47	307.21
0.15	521.85	426.10	369.01	330.06
0.2	567.20	463.10	401.07	358.73

Table 3 also demonstrates the effect of the mass ratio ($\rho A/(m + \rho A)$) on the critical speed. The structural mass decreases as the mass ratio increases for a given fluid density. As a consequence, the ratio of structural flexural stiffness to structural mass will increase. This condition raises the system's natural frequency as well as the critical speed of the fluid flow.

IV. CONCLUSION

In this study, a flutter stability analysis of the fluid-conveying CNT has been conducted. For this purpose, the CNT was modelled as Euler-Bernoulli beams, whereas the effects of shear were neglected. The analysis was carried out using the finite element method, which is combined with the Newmark method time scheme for determining the time response of the system. The flutter that is occurring is indicated by the time response of the system, which is in harmonic form. The velocity of fluid flow that causes flutter is called critical speed. The results showed that the time responses of fluid-conveying CNT are very sensitive to changes in the speed of the fluid flow. Therefore, the determination of the critical speed using time response gives an accurate result. The results also showed that the fluid density has a strong effect on the critical speed of fluid flow and that the increasing mass ratio will increase the critical speed of CNT.

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